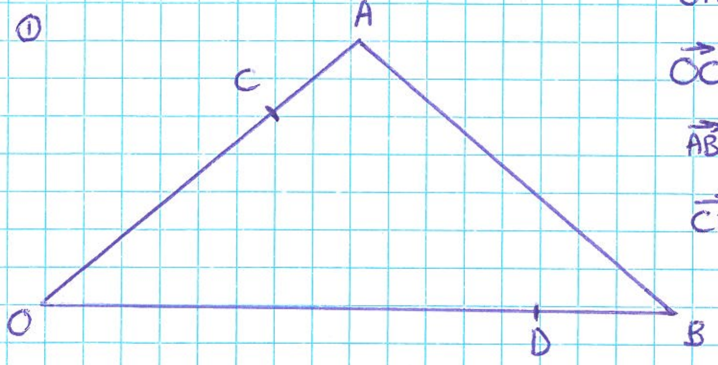
EXERCISE 7A.

①



$\vec{OA} = \underline{a}$ $\quad \vec{OB} = \underline{b}$

$\vec{OC} = h\vec{OA} = h\underline{a}$ $\quad \vec{OD} = h\vec{OB} = h\underline{b}$

$\vec{AB} = \vec{OB} - \vec{OA} = \underline{b} - \underline{a}$

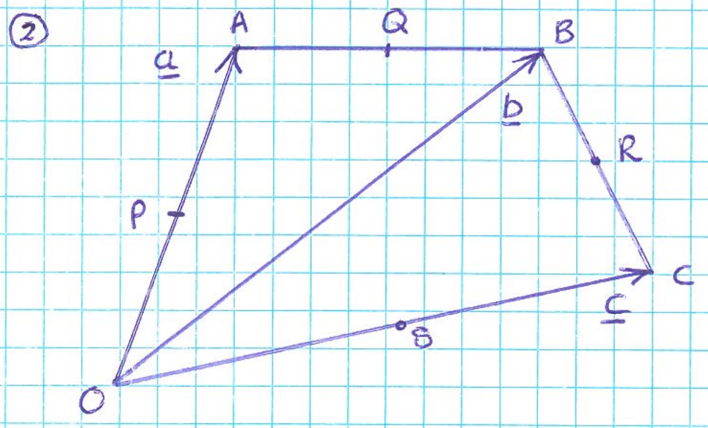
$\vec{CD} = \vec{OD} - \vec{OC} = h\underline{b} - h\underline{a} = h(\underline{b} - \underline{a})$

Therefore, $\vec{CD} = \lambda \vec{AB}$

$\vec{CD}$ is parallel to $\vec{AB}$

---

②



$\vec{OA} = \underline{a}$ $\quad \vec{OB} = \underline{b}$ $\quad \vec{OC} = \underline{c}$

$\vec{AB} = \underline{b} - \underline{a}$ $\quad \vec{BC} = \underline{c} - \underline{b}$

$\vec{PQ} = \frac{1}{2}\vec{OA} + \frac{1}{2}\vec{AB} = \frac{1}{2}\underline{b}$

$\vec{SR} = \frac{1}{2}\vec{OC} - \frac{1}{2}\vec{BC} = \frac{1}{2}\underline{b}$

$\therefore |\vec{PQ}| = |\vec{SR}|$ and $\vec{PQ} \parallel \vec{SR}$

$\vec{PS} = -\frac{1}{2}\vec{OA} + \frac{1}{2}\vec{OC} = -\frac{1}{2}\underline{a} + \frac{1}{2}\underline{c}$

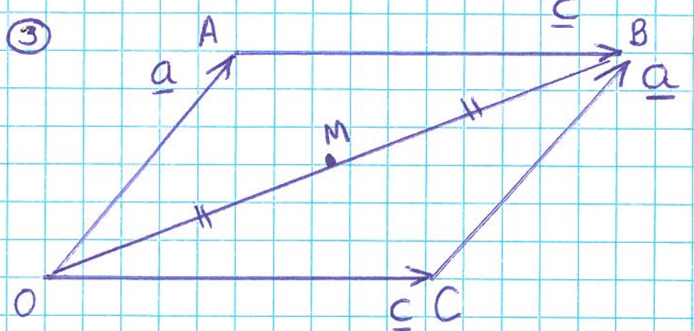
$\vec{QR} = \frac{1}{2}\vec{AB} + \frac{1}{2}\vec{BC} = -\frac{1}{2}\underline{a} + \frac{1}{2}\underline{c}$

$\therefore |\vec{QR}| = |\vec{PS}|$ and $\vec{QR} \parallel \vec{PS}$

$\therefore$ PQRS is a parallelogram;

---

③



$\vec{OB} = \underline{a} + \underline{c}$

$\therefore \vec{OM} = \frac{1}{2}(\underline{a} + \underline{c})$

$\vec{AM} = -\vec{OA} + \vec{OM}$

$= -\underline{a} + \frac{1}{2}(\underline{a} + \underline{c})$

$= -\frac{1}{2}\underline{a} + \frac{1}{2}\underline{c}$

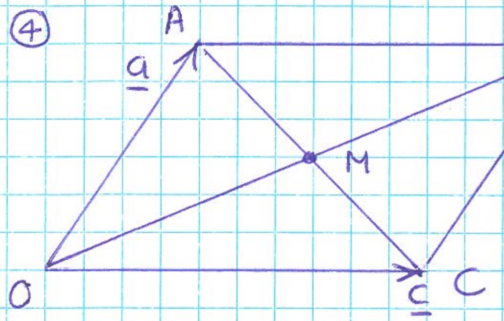
$\vec{AC} = -\underline{a} + \underline{c}$

$\therefore \vec{AM} = \frac{1}{2}\vec{AC}$

$\therefore$ diagonals bisect.

---

(4)



let $\overrightarrow{AM} = h\overrightarrow{AC} = h(-\underline{a}+\underline{c})$

$\overrightarrow{OM} = k\overrightarrow{OB} = k(\underline{a}+\underline{c})$

$\overrightarrow{OM} = \overrightarrow{OA} + \overrightarrow{AM}$

$= \underline{a} + h(-\underline{a}+\underline{c})$

Therefore: $k(a+c) = \underline{a} + h(-\underline{a}+\underline{c})$

$k\underline{a} + k\underline{c} = \underline{a}(1-h) + h\underline{c}$

$\therefore$ equate $\underline{c}$: $h = k$

equate $\underline{a}$: $k = 1-h$

substitute: $k = 1-k$

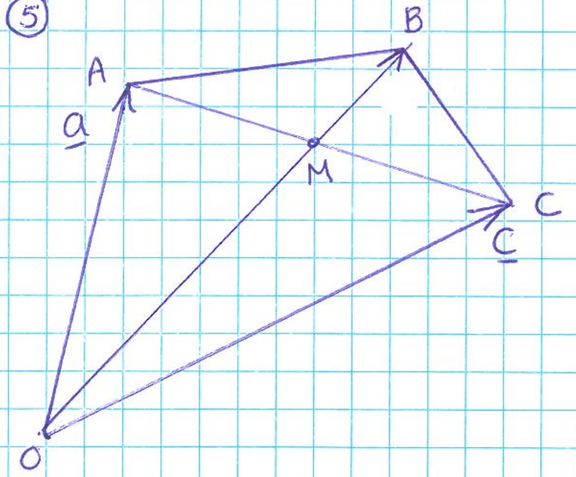
$2k = 1$

$k = 1/2 = h.$

$\therefore$ M is the midpoint of $\overrightarrow{OB}$ and $\overrightarrow{AC}$

---

(5)



Where: $\overrightarrow{AM} = 1/2\,\overrightarrow{AC} = 1/2(-\underline{a}+\underline{c})$

$\therefore \overrightarrow{OM} = \overrightarrow{OA} + \overrightarrow{AM} = \underline{a} + 1/2(-\underline{a}+\underline{c})$

$= 1/2\,\underline{a} + 1/2\,\underline{c}$

$\therefore \overrightarrow{OB} = \underline{a}+\underline{c}$

$\therefore \overrightarrow{OA} = \underline{a}$

$\overrightarrow{CB} = -\overrightarrow{OC} + \overrightarrow{OB} = -\underline{c} + (\underline{a}+\underline{c}) = \underline{a}$

$\overrightarrow{AB} = -\overrightarrow{OA} + \overrightarrow{OB} = -\underline{a} + (\underline{a}+\underline{c}) = \underline{c}$

$\overrightarrow{OC} = \underline{c}$

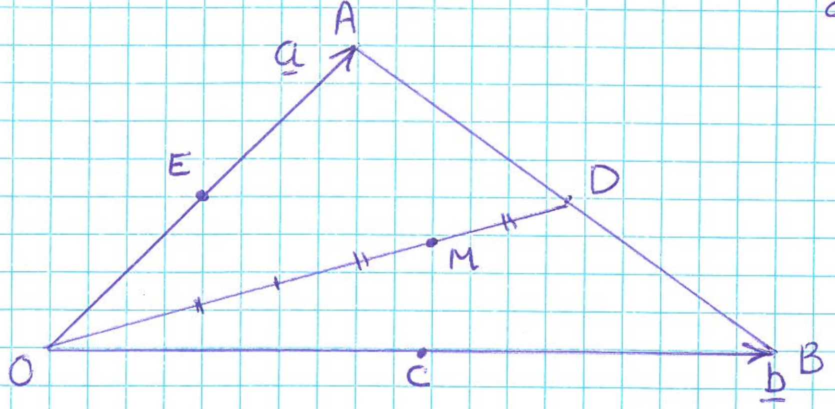
$\therefore \overrightarrow{OA} = \overrightarrow{CB}$ and $\overrightarrow{AB} = \overrightarrow{OC}$ $\therefore$ parallelogram.

The diagonals of a quadrilateral bisect each other $\Leftrightarrow$ the quadrilateral is a parallelogram

---

⑥

MEDIAN – LINE FROM THE VERTEX TO THE MIDPOINT OF THE OPPOSITE SIDE.



a) $\vec{AB} = -\underline{a} + \underline{b}$

$\vec{AC} = -\underline{a} + \tfrac{1}{2}\underline{b}$

$\vec{AD} = -\tfrac{1}{2}\underline{a} + \tfrac{1}{2}\underline{b}$

$\vec{OD} = \vec{OA} + \vec{AD} = \tfrac{1}{2}\underline{a} + \tfrac{1}{2}\underline{b}$

$\vec{OM} = \tfrac{2}{3}\vec{OD} = \tfrac{1}{3}\underline{a} + \tfrac{1}{3}\underline{b}$

$\vec{AM} = -\vec{OA} + \vec{OM} = -\tfrac{2}{3}\underline{a} + \tfrac{1}{3}\underline{b}$

(b) $\vec{AC} = -\underline{a} + \tfrac{1}{2}\underline{b}$

$\vec{AM} = -\vec{OA} + \vec{OM} = -\tfrac{2}{3}\underline{a} + \tfrac{1}{3}\underline{b} = \tfrac{2}{3}(-\underline{a} + \tfrac{1}{2}\underline{b}) = \tfrac{2}{3}\vec{AC}$

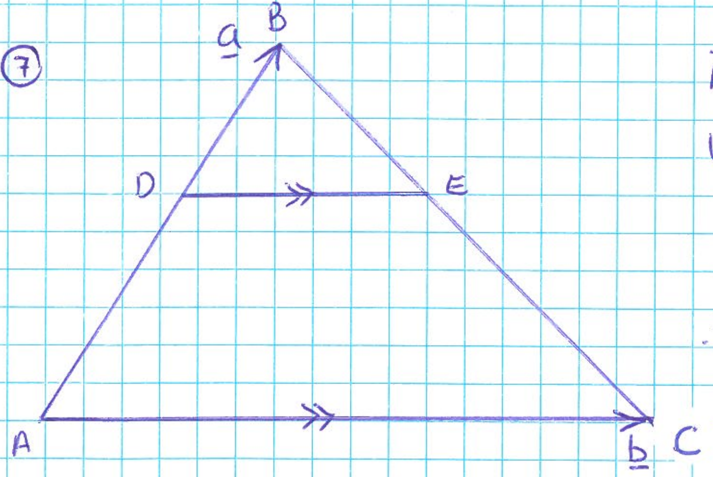
$\therefore AM : MC = 2 : 1$

(c) $\vec{BE} = -\underline{b} + \tfrac{1}{2}\underline{a} = \tfrac{1}{2}\underline{a} - \underline{b}$

$\vec{BM} = -\underline{b} + \vec{OM} = \tfrac{1}{3}\underline{a} - \tfrac{2}{3}\underline{b} = \tfrac{2}{3}(\tfrac{1}{2}\underline{a} - \underline{b}) = \tfrac{2}{3}\vec{BE}$

$\therefore BM : ME = 2 : 1$

⑦



$\vec{AD} = h\vec{AB} = h\underline{a}$

let: $\vec{CE} = k\vec{CB}$

$\vec{DE} = \lambda\vec{AC}$

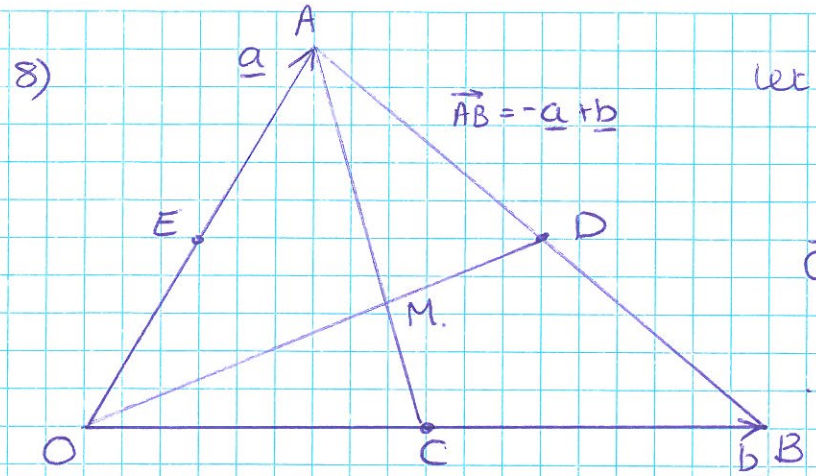
$\therefore -\vec{AD} + \vec{AC} + \vec{CE} = \lambda\vec{AC}$

$-h\underline{a} + \underline{b} + k(\underline{a} - \underline{b}) = \lambda\underline{b}$

$\therefore \underline{a}(k-h) + \underline{b}(1-k) = \lambda\underline{b}$

$\therefore k - h = 0 \qquad \therefore k = h$

8)



let $\vec{OM} = h\vec{OD} = h(\underline{a} + 1/2(-\underline{a} + \underline{b}))$

$\vec{AM} = k\vec{AC} = k(-a + 1/2\,\underline{b})$

$\vec{OM} = \vec{OA} + \vec{AM}$
$= \underline{a} + k(-a + 1/2\,\underline{b})$

$\therefore h(1/2\,\underline{a} + 1/2\,\underline{b}) = \underline{a} + k(-\underline{a} + 1/2\,\underline{b})$

equate $\underline{a}$ $\quad \frac{h}{2} = 1 - k$

equate $\underline{b}$ $\quad \frac{h}{2} = \frac{k}{2}$

$\therefore h = k \quad \frac{k}{2} = 1 - k$

$k = \frac{2}{3}$.

Therefore: $\vec{OM} = 2/3\,\vec{OD} \quad \vec{AM} = 2/3\,\vec{AC}$

$\vec{BE} = -\underline{b} + 1/2\,\underline{a} \quad \vec{BM} = -\underline{b} + \vec{OM} = 1/3\,\underline{a} - 2/3\,\underline{b}$
$= 2/3(-\underline{b} + 1/2\,\underline{a})$

Therefore M lies on $\overline{BE}$.

Now check $\vec{BM}$: let $\vec{BM} = \lambda\vec{BE} = \lambda(-\underline{b} + 1/2\,\underline{a})$

$\vec{OM} = \vec{OB} + \vec{BM} = \underline{b} + \lambda(-\underline{b} + 1/2\,\underline{a})$

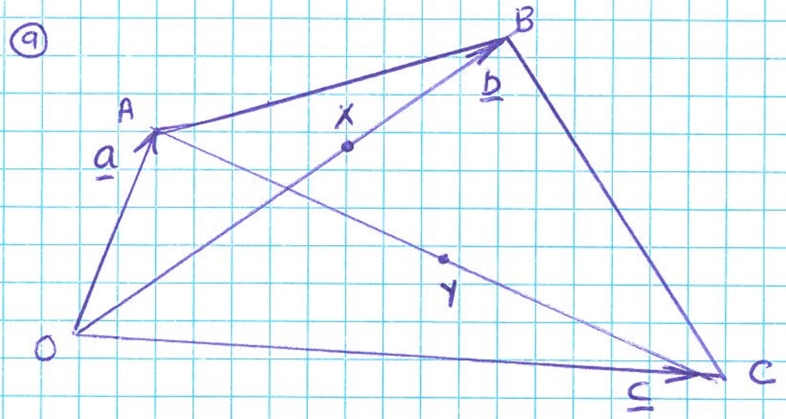
$\therefore \underline{a}(1/2\,\lambda) + b(1 - \lambda) = \underline{a} + 2/3(-\underline{a} + 1/2\,\underline{b})$

$= \underline{a}(1/3) + \underline{b}(1/3)$

$\therefore$ solve to give $\lambda = 2/3 \quad \therefore \vec{BM} = 2/3\,\vec{BE}$

---

(9)



$\vec{OA} = \underline{a}$

$\vec{BA} = -\underline{b} + \underline{a}$

$\vec{OC} = \underline{c}$

$\vec{BC} = -\underline{b} + \underline{c}$

$\vec{XY} = -\vec{OX} + \vec{OA} + \vec{AY}$
$= -\frac{1}{2}(\underline{b}) + \underline{a} + \frac{1}{2}(-\underline{a} + \underline{c})$
$= \frac{1}{2}\underline{a} - \frac{1}{2}\underline{b} + \frac{1}{2}\underline{c}$

Therefore; $\vec{OA} + \vec{BA} + \vec{OC} + \vec{BC} = \underline{a} + (-\underline{b} + \underline{a}) + \underline{c} + (-\underline{b} + \underline{c})$

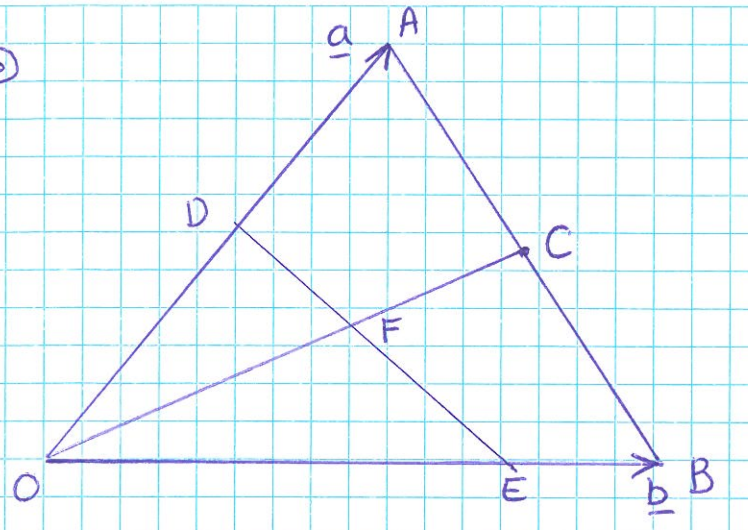
$= 2\underline{a} - 2\underline{b} + 2\underline{c}$

$= 4\left(\frac{1}{2}\underline{a} - \frac{1}{2}\underline{b} + \frac{1}{2}\underline{c}\right)$

$= 4\vec{XY}$

---

(10)



$\overrightarrow{OD} = h\overrightarrow{OA}$

$\overrightarrow{OE} = k\overrightarrow{OB}$

$\overrightarrow{OF} = m\overrightarrow{OC} = m\left(\underline{a} + \tfrac{1}{2}(-\underline{a}+\underline{b})\right)$

$= m\left(\tfrac{1}{2}(\underline{a}+\underline{b})\right)$

a) $\overrightarrow{DF} = -\overrightarrow{OD} + \overrightarrow{OF} = -h\underline{a} + m \times \tfrac{1}{2}(\underline{a}+\underline{b})$

$= \underline{a}\left(-h + \frac{m}{2}\right) + \underline{b}\left(\frac{m}{2}\right)$

b) $\overrightarrow{FE} = -\overrightarrow{OF} + \overrightarrow{OE} = -\frac{m}{2}(\underline{a}+\underline{b}) + k\underline{b}$

$= \underline{a}\left(-\frac{m}{2}\right) + \underline{b}\left(-\frac{m}{2} + k\right)$

If $\overrightarrow{DF} = \overrightarrow{FE}$: equate $\underline{a}$ $\quad -h + \frac{m}{2} = -\frac{m}{2}$

$\therefore m = h.$

equate $\underline{b}$ $\quad \frac{m}{2} = -\frac{m}{2} + k$

$\therefore m = k.$

$\overrightarrow{DE} = \overrightarrow{DF} + \overrightarrow{FE} = \underline{a}(-h) + \underline{b}(k) = h(-\underline{a}+\underline{b})$

$\overrightarrow{AB} = -\underline{a} + \underline{b}$

$\therefore \overrightarrow{DE} = \lambda\overrightarrow{AB} \quad \therefore \overrightarrow{DE} \parallel \overrightarrow{AB}$